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Tunneling of Charged and Magnetized Fermions from the Reissner-Nordström Black Hole with Magnetic Charges

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Abstract The Kerner-Mann fermions tunneling framework is extended to the spin particles with electric and magnetic charges in this paper. We rewrite the electromagnetic field tensor and the Lagrangian of the field corresponding to the source with electric and magnetic charges to redefine an equivalent charge. We only consider the case that the ratio of the electric charge and magnetic charge of the emission is constant and equal to the source. The result shows that when the energy conservation together with the electric charge and magnetic charge conservations are taken into account in the dynamical background space time, the emission rate agrees with the underlying unitary theory and the actual radiation spectrum of charged and magnetized fermions also derivates from the pure thermal one.

Keywords Hawking radiation · Fermions tunneling · Reissner-Nordström black hole

1 Introduction

Thirty years ago, Stephen Hawking [1, 2] first proved that black hole is not fully black but can emit particles in the form of the pure thermal spectrum. He thought Hawking radiation stemmed from the vacuum fluctuations that inside the black hole horizon. That is, when a pair of virtual particles creat spontaneously, the negative energy particle will be absorbed by the hole as its partner tunnels out and materializes in the opposite direction. Nevertheless, the original derivation of Hawking didn't proceed in this way. In 2000 Parikh and Wilczek [3], based on the work of Kraus and Wilczek [4, 5], proposed an intuitively simple but physically rich paradigm to study the tunneling effect. According to their observation, as the particle

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runs out, due to energy conservation, the mass of the black hole will reduce and the radius of the horizon correspondingly will shrink. They also pointed out the tunneling potential barrier was nothing but the radiation itself. Following their work, the massive and massless, charged and uncharged particle have been investigated extensively [6–17]. The same result that Hawking radiation is no longer pure and the unitary theory is satisfied are exhibited. Very recently, in order to confirm the universality of the tunneling picture, Kerner and Mann [18] brought forward an approach to study fermions tunneling. Their work involved the general Dirac equation but not the Newman-Penrose formalism to determine the action of the radiant spin particles. However they only considered the case of the uncharged spin particles and the energy conservation isn't enforced there.

In this paper, we attempt to extend this method to discuss the radiation of charged and magnetized particles from the Reissner-Nordström black hole with magnetic charges. Because the electromagnetic field would couple with the matter field and gravity field, the Dirac equation hence should be modified. The other difficulty here is how to treat the electromagnetic field tensor that stems from the electric and magnetic charges. As the outside of the black hole is an electromagnetic vacuum and the electric together with magnetic charges concentrate on the hole [19], we regard the hole is a conducting sphere. For the sake of simplicity, we assume the rate of the electric charge and magnetic charge is constant and equals to that of the source [20]. In this case, we can redefine an equivalent charge by reconstructing the electromagnetic tensor and Lagrangian density of the field. Then, when we incorporate the self-gravitational interaction and back reaction of the radiant spin particles, we can simplify the consideration of electric charge conservation and magnetic charge conservation to only equivalent charge conservation. In addition, for the static, non-rotating black hole, because the ADM mass is much larger than the Planck mass, we ignore the affect of the radiation on the angular momentum of the black hole. Then for the zero-angular momentum hole, due to the number of the spin up and spin down particles statistically is same, when the spin up particles tunnel out, we regard its partner also do at the same Hawking temperature.

The remainder of this paper are arranged as follows. In Sect. 2, we reconstruct the electromagnetic field tensor and the Lagrangian of the field corresponding to the source with electric charge and magnetic charge to redefine the equivalent charge. In Sect. 3, we discuss the pure thermal spectrum of the spin particles with electric and magnetic charges. Then in Sect. 4, we take the energy and charge conservations into account to discuss the correction spectrum of the charged and magnetized fermions. Finally we present our conclusions in Sect. 5.

2 Maxwell Equation Corresponding to the Source with Electric and Magnetic Charges

Several decades ago, Dirac predicted the existence of the magnetic monopole theoretically. But it was neglected due to the failure to detect such object many years ago. Recent years, the development of gauge theories has shed new light on the ingenious hypothesis though the existence of it is still a mystery. As far as the black hole with electric charge and magnetic charge is concerned, we can introduce the following simplified treatment about the electric and magnetic charge [20].

For a source with electric and magnetic charge, the electromagnetic tensor is

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^{+}_{\mu\nu}, \tag{1}$$

where $G^{+}_{\mu\nu}$ is the Dirac string term. The Maxwell equations read off

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_e u^\mu, \tag{2}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_m u^\mu, \tag{3}$$

where $F^{+\mu\nu}$ is the dual tensor of $F^{\mu\nu}$, ρ_e and ρ_m represent the densities of electric and magnetic charges, while u^μ stand for the 4-velocity. If we define a new real anti-symmetric tensor

$$\tilde{F}^{\mu\nu} = F^{\mu\nu}\cos\beta + F^{+\mu\nu}\sin\beta, \tag{4}$$

where β denotes a real constant angle. Equations (2) and (3) will change as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi(\rho_e\cos\beta + \rho_m\sin\beta)u^\mu, \tag{5}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 4\pi(-\rho_e\sin\beta + \rho_m\cos\beta)u^\mu. \tag{6}$$

Then let

$$\rho_e\cos\beta + \rho_m\sin\beta = \rho_h, \tag{7}$$

$$-\rho_e\sin\beta + \rho_m\cos\beta = 0. \tag{8}$$

Besides yields $\rho_e/\rho_m = \cot\beta$, the Maxwell equations can be simplified as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi\rho_h u^\mu, \tag{9}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 0. \tag{10}$$

where $\tilde{F}_{\mu\nu} = \nabla_\nu\tilde{A}_\mu - \nabla_\mu\tilde{A}_\nu$ and the corresponding general coordinate are

$$\tilde{A}_\mu = (\tilde{A}_0, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3). \tag{11}$$

While $J^\mu = \rho_h u^\mu$ is adopted, (9) also can be written as

$$\frac{\partial}{\partial x^\nu}(\sqrt{-g}\tilde{F}^{\mu\nu}) = 4\pi\sqrt{-g}J^\mu. \tag{12}$$

Obviously, (12) is similar to the Maxwell equation corresponding to the source only with electric charge. Therefore, if we consider the black hole as a conducting sphere while the electric charge and the magnetic charge are concentrated on the black hole with the density rate as $\rho_e/\rho_m = \cot\beta$, we have

$$Q_h^2 = Q_e^2 + Q_m^2, \tag{13}$$

where Q_e and Q_g are the electric charge and magnetic charge of the hole, and Q stand for the equivalent charge corresponding to the density ρ_h. Similarly, the Lagrangian of the field also can be modified as

$$L = -\frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}, \tag{14}$$

where $\tilde{F}^{+\mu\nu}$ is the dual tensor of $\tilde{F}^{\mu\nu}$. Now the Maxwell equation and the Lagrangian corresponding to the source with electric and magnetic charges have been simplified as that only with electric charge, the tunneling of charged and magnetized fermions thus can be treated of.

3 Charged and Magnetized Fermions' Pure Thermal Spectrum

The metric of the Reissner-Nordström black hole with magnetic charges is

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q_e^2 + Q_m^2}{r^2}\right) dt_R^2 + \left(1 - \frac{2M}{r} + \frac{Q_e^2 + Q_m^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{15}$$

Taking into account (13), this line element can be simplified as

$$ds^2 = -f dt_R^2 + f^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{16}$$

where

$$f(r_\pm) = \left(1 - \frac{2M}{r_\pm} + \frac{Q_h^2}{r_\pm^2}\right), \tag{17}$$

M and Q_h denote the mass and equivalent charge, $r_\pm = M \pm \sqrt{M^2 - Q_h^2}$ respectively represent the outer horizon and inner horizon. The electromagnetic vector potential can be rewritten correspondingly as

$$\tilde{A}_\mu = \left(-\frac{Q_h}{r}, 0, 0, 0\right). \tag{18}$$

When a particle approaches to the horizon, its frequency will become smaller for an infinity observer. Thus the geometrical optics limit is reliable at the near-horizon region. In the semi-classical limit, according to the Wentzel-Kramers-Brilloin (WKB) approximation, the relationship between the tunneling rate and the imaginary part of the radiant particle's action can be expressed as

$$\Gamma \sim \mathrm{e}^{-2\,\mathrm{Im}\, I}. \tag{19}$$

Obviously, in order to get the emission probability of the radiation, one should first find its action from the classically forbidden path, which in turn is related to the Boltzmann factor for radiation at Hawking temperature. Until now, there are two ways to accomplish it. One considers the null geodesic equation of the s-wave emitted from the black hole [3] and the other involves the relativistic Hamilton-Jacobi equation [21, 22]. As for the fermions in the electromagnetic field, we have recourse to the Dirac equation

$$\gamma^\mu \left(D_\mu - \frac{i q_h \tilde{A}_\mu}{\hbar}\right)\psi - \frac{im}{\hbar}\psi = 0, \tag{20}$$

where the Greek indices $\mu, \nu = 0, 1, 2, 3$, q_h and m are the equivalent charge and mass of the fermions and

$$D_\mu = \partial_\mu + \Omega_\mu, \quad \Omega_\mu = \frac{1}{2} i \Gamma_\mu^{\alpha\beta} \Sigma_{\alpha\beta}, \quad \Sigma_{\alpha\beta} = \frac{1}{4} i [\gamma^\alpha, \gamma^\beta]. \tag{21}$$

Based on the relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I$, the Gamma matrix in this background can be chosen as $\gamma^t = \gamma^0/\sqrt{f}$, $\gamma^r = \sqrt{f}\gamma^3$, $\gamma^\theta = \gamma^2/r$ and $\gamma^\varphi = \gamma^3/r\sin\theta$ while

$$\gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \tag{22}$$

in which σ^i $(i = 1, 2, 3)$ is general the Pauli Sigma matrixes. The matrix for γ^5 correspondingly takes the form as

$$\gamma^5 = i\gamma^t\gamma^r\gamma^\theta\gamma^\varphi = \frac{i}{r^2\sin\theta}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{23}$$

To solve the (20), we employ the following ansatz

$$\psi_\uparrow(t,r,\theta,\varphi) = \begin{bmatrix} A(t,r,\theta,\varphi)\xi_\uparrow \\ B(t,r,\theta,\varphi)\xi_\uparrow \end{bmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t,r,\theta,\varphi)\right], \tag{24}$$

$$\psi_\downarrow(t,r,\theta,\varphi) = \begin{bmatrix} C(t,r,\theta,\varphi)\xi_\downarrow \\ D(t,r,\theta,\varphi)\xi_\downarrow \end{bmatrix} \exp\left[\frac{i}{\hbar} I_\downarrow(t,r,\theta,\varphi)\right], \tag{25}$$

where $\xi_{\uparrow/\downarrow}$ is the eigenvectors of σ^3, $I_{\uparrow/\downarrow}$ is the action of the radiant spin particles, $A(t,r,\theta,\varphi)$ and $B(t,r,\theta,\varphi)$ correspond to the ingoing and outgoing modes with spin up while $C(t,r,\theta,\varphi)$ and $D(t,r,\theta,\varphi)$ correspond to the outgoing and ingoing modes with spin down. In this paper, we are only interested in the spin up case since the spin down is fully similar to this other than some changes of the sign mathematically. And physically, due to the black hole is static, non-rotating, as the spin up particle runs out, the spin down particle will also do at the same radiation temperature. Inserting (24) into the Dirac Equation, after dividing the exponential term and multiplying $\hbar$, we find to leading order in $\hbar$

$$\frac{iA(\partial_t I_\uparrow - q_h\tilde{A}_t)}{\sqrt{f(r)}} + B\sqrt{f(r)}\partial_r I_\uparrow - Am = 0, \tag{26}$$

$$-\frac{B}{r}\left(\partial_\theta I_\uparrow + \frac{i}{\sin\theta}\partial_\varphi I_\uparrow\right) = 0, \tag{27}$$

$$-\frac{iB(\partial_t I_\uparrow - q_h\tilde{A}_t)}{\sqrt{f(r)}} + A\sqrt{f(r)}\partial_r I_\uparrow - Bm = 0, \tag{28}$$

$$-\frac{A}{r}\left(\partial_\theta I_\uparrow + \frac{i}{\sin\theta}\partial_\varphi I_\uparrow\right) = 0. \tag{29}$$

Solving the action now isn't easy for it is the function of coordinate components. But taking into account the existence of time-like killing vector $(\frac{\partial}{\partial t})^a$, we carry out the following separation variable

$$I_\uparrow = -\omega t + W(r) + J(\varphi,\theta). \tag{30}$$

Then we get

$$-\frac{iA(\omega + q_h\tilde{A}_t)}{\sqrt{f(r)}} + B\sqrt{f(r)}W'(r) - Am = 0, \tag{31}$$

$$-\frac{B}{r}\left(J_\theta + \frac{i}{\sin\theta}J_\varphi\right) = 0, \tag{32}$$

$$\frac{iB(\omega + q_h\tilde{A}_t)}{\sqrt{f(r)}} + A\sqrt{f(r)}W'(r) - Bm = 0, \tag{33}$$

$$-\frac{A}{r}\left(J_\theta + \frac{i}{\sin\theta}J_\varphi\right) = 0. \tag{34}$$

For the azimuthal equation (32) and (34), we find $J_\theta + iJ_\varphi/\sin\theta = 0$ must be satisfied regardless the explicit value of A or B, implying the solutions of $J(\theta,\varphi)$ for the outgoing and ingoing are the same though it may contribute to the imaginary part of the action. On the other hand, for the radial equation (31) and (33), if $m = 0$, there exist two possible solutions that take the form as

$$A = -iB,\, W'(r) = W'_+(r) = \frac{\omega + q_h\tilde{A}_t}{f(r)}, \tag{35}$$

$$A = iB,\, W'(r) = W'_-(r) = -\frac{(\omega + q_h\tilde{A}_t)}{f(r)}, \tag{36}$$

where $W_\pm(r)$ respectively represent the outgoing and incoming solutions of the radial action. And if $m \neq 0$, one can get

$$\left(\frac{A}{B}\right)^2 = \frac{-i(\omega + q_h\tilde{A}_t) + f(r)m}{i(\omega + q_h\tilde{A}_t) + f(r)m}. \tag{37}$$

When the near-horizon approximation is incorporated, which result in $A^2 = -B^2$, we find the same result as the case of massless. Therefore the action of the massive and massless fermions is

$$W_+ = \int \frac{(\omega + q_h\tilde{A}_t)}{f(r)}dr = \frac{i\pi(\omega - \omega_0)(M^2 + M\sqrt{M^2 - Q_h^2} - \frac{1}{2}Q_h^2)}{\sqrt{M^2 - Q_h^2}}, \tag{38}$$

where $\omega_0 = q_h V_0 = q_h Q_h/r_+$. Recalling the semi-classical WKB approximation, when the contributions of the incoming mode is considered, the total tunneling probability can be written as

$$\Gamma \sim \frac{P(\text{out})}{P(\text{in})} = \frac{\exp[-2(\operatorname{Im} W_+ + \operatorname{Im} J)]}{\exp[-2(\operatorname{Im} W_- + \operatorname{Im} J)]} = \mathrm{e}^{-4\operatorname{Im} W_+}. \tag{39}$$

Henceforth we drop the "+" subscript from W_+. The Hawking temperature in this case is

$$T = \frac{\sqrt{M^2 - Q_h^2}}{4\pi(M^2 + M\sqrt{M^2 - Q_h^2} - \frac{1}{2}Q_h^2)}. \tag{40}$$

It is obvious that the radiation spectrum correspond to this temperature is only the pure thermal one.

4 The Correction Spectrum of Fermions with Electric and Magnetic Charges

In fact several years ago, Parikh and Wilczek have proved that the actual radiation spectrum of Hawking radiation isn't the pure thermal spectrum but have some corrections. They meanwhile pointed out that the pure thermal spectrum arises from the energy and charge conservations aren't enforced in the unfixed background. Therefore To precisely picture the Hawking radiation, we have to take the self-gravitational interaction and back reaction of the emitted charged spin particles into account. Fixing the ADM mass and the electric and magnetic charges of black hole, as the particle with a shell of energy ω and equivalent charge q_h tunnels out, the total mass and equivalent charge of the hole would reduce to $M-\omega$ and Q_h-q_h, the radius of the horizon accordingly will shrink. That is, a particle of instantaneous energy ω cross the horizon will effectively "see" a space time metric (16) with the replacement of $M\to M-\omega$ and $Q_h\to Q_h-q_h$. However, due to the restriction of quantum uncertainty principle, it is unnatural to expect that the mass of the black hole can jump. Thus we integrate over ω and equivalent charge q_h in the region $0\to\omega$ and $0\to q_h$. The actual action thus can be rewritten as

$$\begin{aligned}
&\operatorname{Im} W' = \\
&-\pi\int_{0,0}^{\omega,q_h}\frac{[(M-\omega)^2+(M-\omega)\sqrt{(M-\omega)^2-(Q_h-q_h)^2}-\frac{1}{2}(Q_h-q_h)^2]}{\sqrt{(M-\omega)^2-(Q_h-q_h)^2}}d(M-\omega)\\
&-\frac{(Q_h-q_h)[(M-\omega)^2+(M-\omega)\sqrt{(M-\omega)^2-(Q_h-q_h)^2}-\frac{1}{2}(Q_h-q_h)^2]}{r'_+\sqrt{(M-\omega)^2-(Q_h-q_h)^2}}d(Q_h-q_h),
\end{aligned} \tag{41}$$

where $r'_+=r_+(M-\omega,Q_h-q_h)$. Finishing the integration, we get

$$\begin{aligned}
\operatorname{Im} W' &= -\frac{\pi}{4}\{[(M-\omega)^2+(M-\omega)\sqrt{(M-\omega)^2-(Q_e-q_e)^2-(Q_m-q_m)^2}]^2\\
&\quad-[(M-\omega)^2+M\sqrt{M^2-Q_e^2-Q_m^2}]\}\\
&= -\frac{1}{4}\Delta S_{BH},
\end{aligned} \tag{42}$$

where $\Delta S_{BH}=S(M-\omega,Q_e-q_e,Q_m-q_m)-S(M,Q_e,Q_m)$ is the difference of the entropies of the black hole that before and after the radiation with electric and magnetic charges. The emission rate thus can be expressed explicit as

$$\Gamma\sim \mathrm{e}^{\Delta S_{BH}}. \tag{43}$$

This result is consistent with the initial viewpoint of Parikh and Wilczek that the tunneling rate is related to the change of Bekenstein–Hawking entropy, implying the underlying unitary theory is satisfied. To get the correction spectrum of this hole, we can express the difference of the entropies as

$$\Delta S=\frac{dS}{dr_+}\Delta r_+ +\frac{1}{2!}\frac{d^2S}{dr_+^2}(\Delta r_+)^2+\frac{1}{3!}\frac{d^3S}{dr_+^3}(\Delta r_+)^3+\cdots. \tag{44}$$

Employing the expression of event horizon, we find

$$\Delta r_+=\left(\frac{M+\sqrt{M^2-Q_h^2}}{\sqrt{M^2-Q_h^2}}\right)\Delta M-\frac{Q_h\Delta Q_h}{\sqrt{M^2-Q_h^2}}. \tag{45}$$

Subtitling (45) in to (44) and considering the changes of mass and charge of the hole $\Delta M=-\omega$ and $\Delta Q_h=-q_h$, one would get

$$\Gamma\sim\exp(\Delta S)=\exp\left[\frac{(\omega-\omega_0)}{T}\left(1-\frac{1}{2!\beta(\omega-\omega_0)}\frac{d^2S}{dr_+^2}(\Delta r_+)^2\right)\right]. \tag{46}$$

Evidently when the higher order correction that corresponds to the unfixed background space time is considered, the exact emission spectrum of the Reissner-Nordström black hole with magnetic charges derivates from the pure thermal one and one can get the correction.

5 Conclusions

We have extended Kerner-Mann's work on fermions tunneling to the particle with electric and magnetic charges from the charged and magnetized static black hole in the dynamic background space time. We introduced the Dirac equation of charged particles. To simplify our discussion, we redefined an equivalent charge by reconstructing the electromagnetic field tensor and the Lagrangian corresponding to the source with electric and magnetic charges. Then in the background of the Reissner-Nordström black hole with magnetic charges space time, we further considered the self-gravitational interaction and back reaction of the emitted spin particles. Our result shows that when the energy conservation and charge conservation are taken into account, the tunneling probability of charged and magnetized spin particle is related to the change of the Bekenstein–Hawking entropy. This result agrees with that of the scalar particles. To verify our work is reasonable, we can employ the first law of black hole thermodynamics

$$dS_{BH} = \frac{1}{T}dM - V_h dQ_h. \tag{47}$$

As for this black hole

$$V_h = \frac{Q_h}{r}. \tag{48}$$

Incorporating with (7) and (8), (47) can be easily rewritten as

$$dS_{BH} = \frac{1}{T}dM - \frac{V_e}{T}dQ_e - \frac{V_m}{T}dQ_m. \tag{49}$$

According to this result, one also can easily get $\Gamma \sim e^{\Delta S_{BH}}$.

We provided a simplified model to discuss the tunneling of fermions with electric and magnetic charges from the charged and magnetized background space time and our work is also reliable to the tunneling of fermions from the other black holes with electric and magnetic charges.

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